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II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let  $O$  be the intersection of  $MN$ ,  $PQ$ .  $OA=a$ ,  $OB=b$ . Draw  $LD$  parallel to  $OB$ , and let  $D$  be in  $MN$ . Let  $OD=u$ ,  $DL=v$ ,  $AB=c$ ,  $AL=d$ ,  $\angle AOB=\beta$ . Then  $a^2 + b^2 - 2ab\cos\beta = c^2$ ;  $c : d = a : a \pm u$ ;  $c : d = b : v$ .

$$\text{Hence } a = \pm \frac{cu}{d-c}, \quad b = \frac{cv}{d}, \quad \text{and } \frac{c^2 u^2}{(d-c)^2} + \frac{c^2 v^2}{d^2} \mp \frac{2c^2 uv \cos \beta}{d(d-c)} = c^2.$$

$$\therefore \frac{u^2}{(d-c)^2} + \frac{v^2}{d^2} \mp \frac{2uv \cos \beta}{d(d-c)} = 1.$$

$\therefore$  The locus is an ellipse.

III. Solution by A. H. HOLMES, Brunswick, Maine.

Suppose the straight lines  $MN$  and  $PQ$  intersect each other at right angles at  $O$ , and  $AB$  placed between them:  $A$  on  $MN$  and  $B$  on  $PQ$ , and  $L$  a point in  $AB$ . Draw  $LO$ . Put  $AL=b$ ,  $BL=a$ , and  $LO=r$ , and  $LAO=\phi$ ,  $AOL=\theta$ . Then  $b\sin\phi=r\sin\theta$ , and  $a\cos\phi=r\cos\theta$ .

$$\therefore \sin^2\phi = \frac{r^2}{b^2}\sin^2\theta, \text{ and } \cos^2\phi = \frac{r^2}{a^2}\cos^2\theta. \quad \therefore r = \frac{ab}{\sqrt{(a^2\sin^2\theta + b^2\cos^2\theta)}}.$$

Therefore the locus of point  $L$  is an ellipse whose semi-major axis is  $BL$  and whose semi-minor axis is  $AL$ . When  $MN$  and  $PQ$  intersect obliquely at angle  $\psi$  the semi-minor axis would be  $\frac{ab\sin\psi}{\sqrt{(a^2 - b^2\cos^2\psi)}}$ .

Also solved by R. D. Carmichael, and J. Scheffer.

275. Proposed by PROFESSOR WILLIAM HOOVER, Ph. D., Athens, Ohio.

An hyperbola is drawn touching the axes of an ellipse, and the asymptotes of the hyperbola touch the ellipse. Prove that the center of the hyperbola lies on one of the equal conjugate diameters of the ellipse.

Solution by the PROPOSER.

Let  $(x', y')$  be the intersection of the tangents to the ellipse  $a^2y^2 + b^2x^2 - a^2b^2 = 0$  ..... (1); then these tangents being the asymptotes of the hyperbola,  $(x', y')$  is the center of the hyperbola. The equation to the tangents to (1) from  $(x', y')$  is

$$(a^2y^2 + b^2x^2 - a^2b^2)(a^2y'^2 + b^2x'^2 - a^2b^2) = (a^2y'y + b^2x'x - a^2b^2)^2 \text{ ..... (2),}$$

$$\text{or, } (y'^2 - b^2)x^2 + (x'^2 - a^2)y^2 - 2x'y'xy + 2b^2x'x + 2a^2y'y - (a^2y'^2 + b^2x'^2) = 0 \text{ ..... (3).}$$

Now, the equation to the asymptotes of a conic differs from the equation to the conic by a constant only; then adding  $c$  to the left member of (3) we have the equation to the hyperbola.

If now  $y=0$  in this equation to the hyperbola, we have

$$(y'^2 - b^2)x^2 + 2x'x - (a^2y'^2 + b^2x'^2) + c = 0 \dots\dots (4),$$

and the condition that (4) has equal roots, or that the hyperbola touches the  $X$ -axis is given by  $y'^2(a^2b^2 - a^2y'^2 - b^2x'^2) = c(b^2 - y'^2) \dots\dots (5)$ ; and, in a similar way that the curve touches the  $Y$ -axis,  $x'^2(a^2b^2 - a^2y'^2 - b^2x'^2) = c(a^2 - x'^2) \dots\dots (6)$ .  $(5) \div (6)$  gives after reduction,  $a^2y'^2 - b^2x'^2 = 0 \dots\dots (7)$ , showing that  $(x', y')$  is on an equi-conjugate axis of the ellipse.

Also solved by G. W. Greenwood, A. H. Holmes, W. W. Landis, J. Scheffer, and G. B. M. Zerr.

### GROUP THEORY.

12. Proposed by GEORGE H. HALLETT, Ph. D., Assistant Professor of Mathematics, The University of Pennsylvania.

Given  $U_1 = a'$ ,  $V_1 = \beta'$ , and the recursion formulae  $U_y = a'V_{y-1} + a''U_{y-1}$ ,  $V_y = \beta'V_{y-1} + \beta''U_{y-1}$ . Find expressions for  $U_y$ ,  $V_y$  in terms of the coefficients  $a'$ ,  $a''$ ,  $\beta'$ ,  $\beta''$ .

Solution by PROFESSOR JAMES BYRNIE SHAW, The James Milliken University, Decatur, Ill.

By eliminating  $V$  we find that  $U_n$  is the coefficient of  $x^n$  in the expansion of

$$\frac{a'x}{1 - (a'' + \beta')x + (a''\beta' - a'\beta'')x^2}.$$

Likewise we find that  $V_n$  is the coefficient of  $x^n$  in the expansion of

$$\frac{1 - a''x}{1 - (a'' + \beta')x + (a''\beta' - a'\beta'')x^2}.$$

We may state the result as follows: Let  $\cos\theta = \frac{1}{2} \cdot \frac{a'' + \beta'}{T}$  where  $T^2 = \begin{vmatrix} a'' & a' \\ \beta'' & \beta' \end{vmatrix}$ .

Then  $V_n = a' \cdot T^{n-1} \cdot \frac{\sin n\theta}{\sin\theta}$ , and  $V_n = T^n \cdot \frac{\sin(n+1)\theta}{\sin\theta} - a'' T^{n-1} \frac{\sin n\theta}{\sin\theta}$ . These latter forms are easily verified by mathematical induction. The well-known formulae for  $\frac{\sin n\theta}{\sin\theta}$  give  $U_n$  and  $V_n$  in terms of the coefficients directly, and free from irrationalities.

13. Proposed by O. E. GLENN, Ph. D., Springfield, Mo.

The order of the linear homogeneous group in  $n$  letters is  $(p^n - 1)(p^n - p) \dots\dots (p^n - p^{n-1})$ . Two proofs are given in Burnside's *Finite Groups*. Give other proofs.

Solution by the PROPOSER.

The linear homogeneous group is known to be equivalent to the group of isomorphisms of the abelian group  $H_{p^n} = [P_1, P_2, \dots, P_n]$  of type  $[1 \ 1 \ 1 \dots, ]$ ,